10 May 51

NACA TN 2350

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2350

ON THE SECOND-ORDER TUNNEL-WALL-CONSTRICTION CORRECTIONS IN TWO-DIMENSIONAL COMPRESSIBLE FLOW

By E. B. Klunker and Keith C. Harder

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
April 1951

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2350

ON THE SECOND-ORDER TUNNEL-WALL-CONSTRICTION CORRECTIONS IN TWO-DIMENSIONAL COMPRESSIBLE FLOW

By E. B. Klunker and Keith C. Harder

SUMMARY

Solutions of the first- and second-order Prandtl-Busemann iteration equations are obtained for the flow past thin, sharp-nose, symmetric, two-dimensional bodies in closed channels. With the use of these solutions an expression is derived for the tunnel-wall interference. The tunnel-wall correction for a parabolic-arc airfoil is calculated to indicate the effects of compressibility, ratio of the tunnel height to the airfoil chord, and airfoil thickness coefficient. It appears that, for cases where the tunnel-wall corrections are significant, both the second-order effects and the variation of the correction along the chord should be considered.

INTRODUCTION

The use of wind tunnels for solving many aerodynamic problems in the high-subsonic speed range makes it desirable to reexamine the question of tunnel-wall corrections. The general problem of tunnel-wall interference in incompressible-flow fields has been treated in reference 1. There the various types of interference are described and equations are presented which permit the correction of wind-tunnel data to free-flight conditions. These results may be carried over to include the first-order effects of compressibility with the aid of the familiar Prandtl-Glauert rule. (See reference 2, for example.) It is well-known, however, that the linearized compressible-flow relations in general do not describe the phenomena accurately at high-subsonic Mach numbers.

The exact analytical solution for the compressible flow past an arbitrary closed body has not been formulated; however, certain approximate methods prove useful for studying flow problems. The Rayleigh-Janzen method, in which the initial step is the complete incompressible-flow solution, has been used frequently for obtaining approximate solutions to aerodynamic problems. In reference 3 the flow past a circular cylinder in a tunnel has been obtained by this procedure. The Rayleigh-Janzen method is best suited, however, for obtaining flows past relatively

blunt profiles at low Mach numbers; hence, this method is not suitable for evaluating the tunnel-wall interference for thin profiles at high-subsonic Mach numbers. Solutions of flow problems by relaxation methods have proved useful in many instances; this approach has been employed in references 4 and 5 to evaluate the tunnel-wall-constriction corrections for the NACA 0012 airfoil and the Kaplan bump, respectively. Although accurate solutions can be obtained by this method, lengthy numerical calculations are involved and the result is confined to the specific profile and channel configuration treated.

The Prandtl-Busemann small-disturbance iteration method has been found most useful in the evaluation of the flow over thin bodies in an infinite stream. This method therefore is employed herein to evaluate the second-order tunnel-wall corrections for the flow past thin symmetric bodies at zero incidence. The third- and higher-order solutions can also be obtained by this method although the labor involved becomes prohibitive.

The wall-interference effects are often calculated from a suitable system of images. Other approaches are possible, however, and in many instances prove more convenient. In particular, an integral representation proves useful for evaluating some interference effects. This representation is employed herein to evaluate the wall interference (constriction effects) at subsonic speeds for a thin, two-dimensional, symmetric, sharp-nose body in a tunnel. The method is closely related to the familiar source-sink concepts for evaluating the flow over a thin body in an unbounded stream. The extension of the source-sink concepts to the solution of the second-order Prandtl-Busemann iteration equations has been previously discussed in reference 6. The method of solution of the interference problem to the second order thus parallels this work. As an example of the use of the equations developed herein, the wall-interference effects are evaluated for a symmetric parabolic-arc airfoil in a two-dimensional channel.

ANALYSIS

The exact nonlinear equation governing the two-dimensional flow of a compressible fluid for the velocity potential Φ' is

$$\left(1 - \frac{u'^2}{a^2}\right)\Phi'_{x'x'} - 2\,\frac{u'v'}{a^2}\,\Phi'_{x'y'} + \left(1 - \frac{v'^2}{a^2}\right)\Phi'_{y'y'} = 0$$

where

x', y'	rectangular Cartesian coordinates in the flow plane
u', v'	velocity components along the x'- and y'-axes, respectively
a	local speed of sound

and the subscripts x' and y' denote partial differentiation with respect to these designated variables. With the introduction of a characteristic length $c/2$, where c is the chord, and the undisturbed stream velocity U as the unit of velocity, the potential equation may be written in the nondimensional form

$$\left(1 - \frac{u^2}{a^2}\right)\Phi_{xx} - 2\,\frac{uv}{a^2}\,\Phi_{xy} + \left(1 - \frac{v^2}{a^2}\right)\Phi_{yy} = 0 \tag{1}$$

where $x = \frac{2x'}{c}$, $y = \frac{2y'}{c}$, $u = \frac{u'}{U}$, $v = \frac{v'}{U}$, and $\Phi = \frac{2\Phi'}{Uc}$.

The Prandtl-Busemann iteration equations are developed from the nonlinear potential equation (1) (see reference 7, for example) by assuming that Φ may be expanded in the form $\Phi = x + \phi_1 + \phi_2 + \ldots$ where ϕ_{n+1} and its derivatives are small compared to ϕ_n and ϕ_n is of the order t^n where t is the thickness coefficient of the airfoil. With these assumptions the first two equations of the Prandtl-Busemann iteration method are

$$\beta^2\phi_{1xx} + \phi_{1yy} = 0 \tag{2a}$$

$$\beta^2\phi_{2xx} + \phi_{2yy} = 2M_\infty^2\left[(1 + \sigma)\beta^2\phi_{1xx}\phi_{1x} + \phi_{1xy}\phi_{1y}\right] \tag{2b}$$

where $\sigma = \frac{\gamma + 1}{2}\,\frac{M_\infty^2}{\beta^2}$, $\beta^2 = 1 - M_\infty^2$, M_∞ is the stream Mach number, and γ is the ratio of specific heats at constant pressure and constant volume.

The constraint of the tunnel walls, or the tunnel-wall correction, is expressed as the difference between the velocity at any point in the flow in the tunnel and the velocity at the same point in an unbounded stream. For aeronautical applications this velocity increment is of interest primarily on the surface of the body, that is, for correcting the surface pressures on a body in a wind tunnel to free-stream conditions. This tunnel correction on the surface of the body is found to the second order by solving equations (2) subject to proper boundary conditions.

Throughout this paper a bar over a quantity (for example, $\bar{\Phi}$) represents the quantity in the channel whereas the same quantity without the bar (for example, Φ) denotes the quantity in an unbounded stream. The differential equations (2) are, of course, the same for the flow in the channel and in a free stream.

Boundary Conditions

Let the equations

$$Y = tY_1(x) \qquad (-1 < x < 1) \tag{3a}$$

and

$$Y = 0 \qquad (x \geqq 1;\ x \leqq -1) \tag{3b}$$

define a thin symmetric body of thickness coefficient t lying on the x-axis between -1 and 1. Then the boundary conditions for the flow over the body in an unbounded stream are:

At infinity

$$\Phi_x = 1 \quad , \quad \Phi_y = 0 \tag{4a}$$

and on the body

$$\Phi_y(x,Y) = Y'\Phi_x(x,Y) \tag{4b}$$

where Y' denotes the slope dY/dx. For the flow over the body defined by equations (3) in a tunnel (fig. 1) whose walls are at $y' = \pm h'$ (or $y = \pm h$) the boundary conditions are:

At the tunnel wall

$$\bar{\Phi}_y(x,h) = 0 \tag{5a}$$

on the body

$$\bar{\Phi}_y(x,Y) = Y'\bar{\Phi}_x(x,Y) \tag{5b}$$

and at $x \rightarrow \pm\infty$

$$\bar{\Phi}_x = 1 \quad , \quad \bar{\Phi}_y = 0 \tag{5c}$$

The boundary conditions for ϕ_1 and ϕ_2 to the order t and t^2, respectively, are:

At infinity

$$\phi_{1x} = \phi_{1y} = 0 \tag{6a}$$

on the body

$$\phi_{1y}(x,0) = tY' \tag{6b}$$

and at infinity

$$\phi_{2x} = \phi_{2y} = 0 \tag{7a}$$

on the body

$$\phi_{2y}(x,0) = tY_1'\phi_{1x}(x,0) + \beta^2 tY_1\phi_{1xx}(x,0) \tag{7b}$$

Similarly, the boundary conditions for $\bar{\phi}_1$ and $\bar{\phi}_2$ to the order t and t^2, respectively, are:

As $x \rightarrow \pm\infty$

$$\bar{\phi}_{1x} = \bar{\phi}_{1y} = 0 \tag{8a}$$

at the wall $y = h$

$$\bar{\phi}_{1y}(x,h) = 0 \tag{8b}$$

on the body

$$\bar{\phi}_{1y}(x,0) = tY_1' \tag{8c}$$

and as $x \rightarrow \pm\infty$

$$\bar{\phi}_{2x} = \bar{\phi}_{2y} = 0 \tag{9a}$$

at the wall $y = h$

$$\bar{\phi}_{2y}(x,h) = 0 \tag{9b}$$

on the body

$$\bar{\phi}_{2y}(x,0) = tY_1'\bar{\phi}_{1x}(x,0) + \beta^2 tY_1\bar{\phi}_{1xx}(x,0) \tag{9c}$$

The solution of the nonhomogeneous second-order equation (2b) may be expressed as the sum of a particular integral $\psi_2(x,y)$ (or $\bar{\Psi}_2(x,y)$) and a function $\varphi_2(x,y)$ (or $\bar{\varphi}_2(x,y)$) satisfying the homogeneous equation

$$\beta^2\varphi_{2xx} + \varphi_{2yy} = 0 \tag{10}$$

The particular integral for equation (2b) (reference 8) is

$$\psi_2(x,y) = M_\infty^2\phi_{1x}\left[\left(1 + \frac{\sigma}{2}\right)\phi_1 - \frac{\sigma}{2}\,y\phi_{1y}\right] \tag{11}$$

Then from equations (7) and (9) the boundary conditions for φ_2 and $\overline{\varphi}_2$ are:

At infinity

$$\varphi_{2x} = \varphi_{2y} = 0$$

on the body

$$\varphi_{2y}(x,0) = F(x)$$

where

$$F(x) \equiv \beta^2 t Y_1{}' \phi_{1x}(x,0) + \beta^2 t Y_1 \phi_{1xx}(x,0) - M_\infty{}^2\left(1 + \frac{\sigma}{2}\right)t\phi_1(x,0)\phi_{1xy}(x,0)$$

and at $x \to \pm\infty$

$$\overline{\varphi}_{2x} = \overline{\varphi}_{2y} = 0 \tag{12a}$$

at the wall $y = h$

$$\overline{\varphi}_{2y}(x,h) = \overline{f}(x) \tag{12b}$$

on the body

$$\overline{\varphi}_{2y}(x,0) = \overline{F}(x) \tag{12c}$$

where

$$\overline{F}(x) \equiv \beta^2 t Y_1{}' \overline{\phi}_{1x}(x,0) + \beta^2 t Y_1 \overline{\phi}_{1xx}(x,0) - M_\infty{}^2\left(1 + \frac{\sigma}{2}\right)t\overline{\phi}_1(x,0)\ \overline{\phi}_{1xy}(x,0)$$

$$\overline{f}(x) \equiv M_\infty{}^2\beta^2\ \frac{\sigma}{2}\ h\overline{\phi}_{1x}(x,h)\overline{\phi}_{1xx}(x,h)$$

The tunnel-wall interference to the second order then is found from the solution of equations (2) subject to the given boundary conditions.

Solution of the Boundary-Value Problem

First-order solution.- A solution of the first of equations (2) for $\bar{\phi}_1$, satisfying the first two boundary conditions of equations (8), is, for $y \geqq 0$,

$$\bar{\phi}_1(x,y) = -\frac{1}{\pi}\int_0^\infty \frac{d\alpha}{\alpha}\int_{-\infty}^\infty g(\xi)\,\frac{\cosh \beta\alpha(y-h)}{\sinh \beta\alpha h}\cos \alpha(x-\xi)\,d\xi$$

from which

$$\bar{\phi}_{1y}(x,0) = \frac{\beta}{\pi}\int_0^\infty d\alpha\int_{-\infty}^\infty g(\xi)\cos \alpha(x-\xi)\,d\xi \tag{13}$$

The body slope tY_1' may be represented by its Fourier integral as

$$tY_1'(x) = \frac{t}{\pi}\int_0^\infty d\alpha\int_{-1}^1 Y_1'(\xi)\cos \alpha(x-\xi)\,d\xi \tag{14}$$

From equations (13) and (14) the boundary condition for tangential flow (equation (8c)) on the body is satisfied by choosing $g(\xi) = \frac{t}{\beta}Y_1'(\xi)$. Thus the solution for $\bar{\phi}_1(x,y)$ is

$$\bar{\phi}_1(x,y) = -\frac{t}{\beta\pi}\int_0^\infty \frac{d\alpha}{\alpha}\int_{-1}^1 Y_1'(\xi)\,\frac{\cosh \beta\alpha(y-h)}{\sinh \beta\alpha h}\cos \alpha(x-\xi)\,d\xi$$

Inverting the order of integration and integrating with respect to α gives

$$\bar{\phi}_1(x,y) = \frac{t}{\beta\pi}\int_{-1}^1 Y_1'(\xi)\log_e\left\{\frac{1}{2\lambda^2}\left[\cosh 2\lambda(x-\xi) - \cos 2\lambda\beta y\right]\right\}d\xi \tag{15}$$

where $\lambda = \frac{\pi}{2\beta h}$. The solution $\phi_1(x,y)$ satisfying the boundary conditions given by equations (6) may be found from equation (15) by letting h approach infinity. Thus

$$\phi_1(x,y) = \lim_{h \to \infty} \overline{\phi}_1(x,y) = \frac{t}{\beta\pi}\int_{-1}^{1} Y_1'(\xi)\log_e\sqrt{(x-\xi)^2+\beta^2y^2}\,d\xi \qquad (16)$$

which is the well-known solution for the flow past a symmetric body in an unbounded stream by a distribution of sources.

With $\Delta u_1 = \overline{\phi}_{1x}(x,0) - \phi_{1x}(x,0)$, the difference in velocity on the body to the first order is

$$\Delta u_1 = \frac{t}{\beta\pi}\int_{-1}^{1} Y_1'(\xi)\left[\lambda \coth \lambda(x-\xi) - \frac{1}{x-\xi}\right]d\xi \qquad (17)$$

The first-order tunnel-wall correction to the x-component of velocity on the body is given by equation (17). This solution was obtained by considering the flow in the upper half-plane; the solution in the lower half-plane is known by symmetry.

Second-order solution.- The second-order solution for the flow over a symmetric body by an extension of the concepts of source-sink distributions has been discussed in reference 6. The second-order solution ϕ_2 is given by $\phi_2 = \varphi_2 + \psi_2$ where φ_2 satisfies the homogeneous equation (10) and ψ_2 is the particular integral given by equation (11). The form of the differential equation and boundary conditions for φ_2 are the same as for ϕ_1. Thus the solution for ϕ_2 may be written immediately as

$$\phi_2 = \frac{1}{\beta\pi}\int_{-1}^{1} F(\xi)\log_e\sqrt{(x-\xi)^2+\beta^2y^2}\,d\xi + \psi_2$$

and

$$\phi_{2x}(x,0) = \frac{1}{\beta\pi}\int_{-1}^{1} F(\xi)\,\frac{d\xi}{x-\xi} + M_\infty^2\left(1+\frac{g}{2}\right)\left[\phi_1(x,0)\phi_{1xx}(x,0) + \phi_{1x}^2(x,0)\right] \qquad (18)$$

(It should be noted that the second-order solutions given here are not valid for blunt-nose profiles since for these cases the particular integral introduces singularities which cannot be canceled by a source distribution.)

The second-order solution for the flow over a symmetric body in a channel may be found in a manner analogous to that for ϕ_1. Consider the expression

$$\overline{\varphi}_2 = -\frac{1}{\beta\pi}\int_0^\infty \frac{d\alpha}{\alpha}\int_{-1}^{1} \overline{F}(\xi)\,\frac{\cosh\beta\alpha(y-h)}{\sinh\beta\alpha h}\cos\alpha(x-\xi)\,d\xi\,+$$

$$\frac{1}{\beta\pi}\int_0^\infty \frac{d\alpha}{\alpha}\int_{-\infty}^{\infty} \overline{f}(\xi)\,\frac{\cosh\beta\alpha y}{\sinh\beta\alpha h}\cos\alpha(x-\xi)\,d\xi \tag{19}$$

which is a solution of the homogeneous differential equation (10). Since, from equation (19),

$$\overline{\varphi}_{2y}(x,0) = \frac{1}{\pi}\int_0^\infty d\alpha\int_{-1}^{1}\overline{F}(\xi)\cos\alpha(x-\xi)\,d\xi$$

and

$$\overline{\varphi}_{2y}(x,h) = \frac{1}{\pi}\int_0^\infty d\alpha\int_{-\infty}^{\infty}\overline{f}(\xi)\cos\alpha(x-\xi)\,d\xi$$

which equal $\overline{F}(x)$ and $\overline{f}(x)$, respectively, by the Fourier integral theorem, the boundary conditions of equations (12) on the body and at the wall are satisfied. Inverting the order of integration of equation (19) and integrating with respect to α gives

$$\overline{\varphi}_2 = \frac{1}{2\beta\pi}\int_{-1}^{1}\overline{F}(\xi)\log_e\left\{\frac{1}{2\lambda^2}\left[\cosh 2\lambda(x-\xi) - \cos 2\lambda\beta y\right]\right\}d\xi\,-$$
$$\frac{1}{2\beta\pi}\int_{-\infty}^{\infty}\overline{f}(\xi)\log_e\left\{\frac{1}{2\lambda^2}\left[\cosh 2\lambda(x-\xi) + \cos 2\lambda\beta y\right]\right\}d\xi$$

from which the boundary conditions at infinity may be shown to be satisfied.

The quantity $\overline{\phi}_{2x}(x,0)$ is found after some reduction as

$$\overline{\phi}_{2x}(x,0) = \frac{\lambda}{\beta\pi}\int_{-1}^{1} \overline{F}(\xi) \coth \lambda(x - \xi)\, d\xi -$$

$$\frac{\lambda}{\beta\pi}\int_{-\infty}^{\infty} \overline{f}(\xi) \tanh \lambda(x - \xi)\, d\xi + \overline{\Psi}_{2x}(x,0) \qquad (20)$$

The second-order correction to the velocity on the surface of a body in a wind tunnel is

$$\Delta u = \Delta u_1 + \Delta u_2 + O(t^3)$$

where

$$\Delta u_1 = \overline{\phi}_{1x}(x,0) - \phi_{1x}(x,0)$$

$$\Delta u_2 = \overline{\phi}_{2x}(x,0) - \phi_{2x}(x,0)$$

or from equations (17), (18), and (20)

$$\Delta u = \frac{t}{\beta\pi}\int_{-1}^{1} Y_1'(\xi)\left[\lambda \coth \lambda(x - \xi) - \frac{1}{x - \xi}\right]d\xi +$$

$$\frac{\lambda}{\beta\pi}\int_{-1}^{1} \overline{F}(\xi) \coth \lambda(x - \xi)\, d\xi - \frac{\lambda}{\beta\pi}\int_{-\infty}^{\infty} \overline{f}(\xi) \tanh \lambda(x - \xi)\, d\xi -$$

$$\frac{1}{\beta\pi}\int_{-1}^{1} F(\xi)\, \frac{d\xi}{x - \xi} +$$

$$M_\infty^2\left(1 + \frac{\sigma}{2}\right)\left[\overline{\phi}_1(x,0)\overline{\phi}_{1xx}(x,0) + \overline{\phi}_{1x}^{\,2}(x,0) - \phi_1(x,0)\phi_{1xx}(x,0) -\right.$$

$$\left.\phi_{1x}^{\,2}(x,0)\right] + O(t^3) \qquad (21)$$

Throughout this paper the Cauchy principal value of all improper integrals is to be taken.

Discussion of the Equations

The tunnel-wall-interference effects are often evaluated by considering a suitable system of images. (See references 1 and 2, for example.) The first-order equations developed previously are consistent with this approach. An expansion for $\coth \lambda(x - \xi)$ (see reference 9, for example) is

$$\coth \lambda(x - \xi) = \frac{1}{\lambda(x - \xi)} + 2 \sum_{n=1}^{\infty} \frac{\lambda(x - \xi)}{n^2\pi^2 + \lambda^2(x - \xi)^2}$$

Then, from equation (17),

$$\Delta u_1 = \frac{2\lambda t}{\beta\pi} \sum_{n=1}^{\infty} \int_{-1}^{1} Y_1'(\xi) \frac{\lambda(x - \xi)}{\lambda^2(x - \xi)^2 + n^2\pi^2} d\xi$$

which is recognized as the influence of source-sink images located at $y = \frac{n\pi}{\lambda}$. The series appearing in the expression for Δu_1 is uniformly convergent; hence the term-by-term integration is valid.

The calculation of the tunnel-wall interference for a given profile requires the evaluation of several integrals. The integrals occurring in equation (21) are not easily evaluated for most profiles and it is preferable to expand the integrands in the form of a series. The series given previously, however, is not the most useful. The expansions

$$\left.\begin{aligned} \lambda \coth \lambda(x - \xi) &= \frac{1}{x - \xi} + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^{2n}\lambda^{2n}B_n}{(2n)!} (x - \xi)^{2n-1} \\ &\qquad \left(\lambda^2(x - \xi)^2 < \pi^2\right) \\ \tanh \lambda(x - \xi) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^{2n}(2^{2n} - 1)\lambda^{2n-1}B_n}{(2n)!} (x - \xi)^{2n-1} \\ &\qquad \left(\lambda^2(x - \xi)^2 < \frac{\pi^2}{4}\right) \end{aligned}\right\} \quad (22)$$

(see reference 10, for example), where the coefficients B_n are the Bernoulli numbers, are better suited for calculation since the integrations are more easily performed. Moreover these alternating series converge rapidly and in most cases only a few terms need be retained.

With the use of the relations (22) the first-order correction to the velocity on the body is determined from equation (17) as

$$\Delta u_1 = \frac{t}{\beta\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^{2n}\lambda^{2n}B_n}{(2n)!}\int_{-1}^{1}Y_1'(\xi)(x-\xi)^{2n-1}d\xi$$

$$\left(\lambda^2(x-\xi)^2 < \pi^2\right) \quad (23)$$

or, after integration by parts,

$$\Delta u_1 = \frac{t}{\beta\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^{2n}\lambda^{2n}(2n-1)B_n}{(2n)!}\int_{-1}^{1}Y_1(\xi)(x-\xi)^{2n-2}d\xi$$

$$\left(\lambda^2(x-\xi)^2 < \pi^2\right)$$

Thus the first term is proportional to the area of the body and the remaining terms are in a form convenient for graphical integration.

Similarly, to the second order, the correction to the surface velocity is

$$\Delta u = \frac{t}{\beta\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^{2n}\lambda^{2n}B_n}{(2n)!}\int_{-1}^{1}Y_1'(\xi)(x-\xi)^{2n-1}d\xi +$$

$$\frac{1}{\beta\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^{2n}\lambda^{2n}B_n}{(2n)!}\int_{-1}^{1}\overline{F}(\xi)(x-\xi)^{2n-1}d\xi + \frac{1}{\beta\pi}\int_{-1}^{1}\frac{\overline{F}(\xi)-F(\xi)}{x-\xi}d\xi +$$

$$\frac{1}{\beta\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2^{2n}(2^{2n}-1)\lambda^{2n}B_n}{(2n)!}\int_{-1}^{1}\overline{f}(\xi)(x-\xi)^{2n-1}d\xi +$$

$$M_\infty^2\left(1+\frac{\sigma}{2}\right)\Big[\overline{\phi}_1(x,0)\overline{\phi}_{1xx}(x,0) + \overline{\phi}_{1x}^2(x,0) - \phi_1(x,0)\phi_{1xx}(x,0) -$$

$$\phi_{1x}^2(x,0)\Big] + O(t^3)$$

$$\left(\lambda^2(x-\xi)^2 < \frac{\pi^2}{4}\right) \quad (24)$$

The quantities $\overline{\phi}_1$, ϕ_1, and their derivatives required to evaluate the functions $\overline{F}$, F, and $\overline{f}$ as well as the particular integral may be found from the equations (15) and (16). These quantities can be expressed in series form with the use of the relations (22).

The range of integration for the integral containing $\overline{f}(\xi)$ should extend from $-\infty$ to ∞. The series employed, however, does not converge over this entire range. The contribution of this integral is small outside the range -1 to 1; hence, with good approximation, the integration may be taken from -1 to 1. In some cases the contribution of the integral involving $\overline{f}(\xi)$ is so small that it may be neglected altogether. In this case, equation (24) converges for $\frac{\pi}{2\beta h}\left|x - \xi\right| < \pi$. Since the maximum value of $x - \xi$ is the chord, the series form for the correction is applicable to cases where the chord is less than $\sqrt{1 - M_\infty^2}$ times the tunnel height. When the integral involving $\overline{f}(\xi)$ is considered, the series solution is applicable to cases where the chord is less than $\frac{1}{2}\sqrt{1 - M_\infty^2}$ times the tunnel height.

The constriction effects for an open tunnel may be found by an analysis similar to that given for the closed tunnel. Here the boundary condition of zero normal velocity at the wall is replaced by the condition of constant pressure along the jet boundary. To the first order the jet boundary is located at $y' = h'$. However, for the second-order solution the boundary condition must be satisfied on the jet boundary. The location of this boundary for the second-order solution may be determined from the first-order velocities at $y' = h'$. The fact that the ordinate of the jet boundary is not a constant and is initially unknown for any iteration higher than the first makes the solution of this problem more tedious than that for the closed tunnel.

TUNNEL-WALL CORRECTIONS FOR A SYMMETRIC PARABOLIC-ARC AIRFOIL

As an example of the use of the equations developed herein, the tunnel-wall corrections are evaluated for a parabolic-arc airfoil. This profile proves convenient since the first-order wall corrections can be obtained in terms of tabulated functions. Thus the rapidity of convergence of the alternating-series form of solution is readily ascertained by comparison with this solution.

The parabolic-arc airfoil is defined by the equation

$$Y(x) = tY_1(x) = t(1 - x^2) \qquad (-1 < x < 1)$$

$$= 0 \qquad (x > 1;\ x < -1)$$

From equation (16) the u-component of velocity on the airfoil in an unbounded stream is given to the first order as

$$u_1 = \frac{t}{\beta\pi} \mathcal{P}\int_{-1}^{1} \frac{Y_1'(\xi)}{x - \xi} d\xi$$

$$= -\frac{2t}{\beta\pi} \mathcal{P}\int_{-1}^{1} \frac{\xi}{x - \xi} d\xi$$

$$= \frac{2t}{\beta\pi}\left(2 + x \log_e \left|\frac{1 - x}{1 + x}\right|\right) \tag{25}$$

where $\mathcal{P}$ denotes the Cauchy principal value. From equation (15) the $\bar{u}$-component of velocity on the airfoil in the tunnel is

$$\bar{u}_1 = \frac{\lambda t}{\beta\pi} \mathcal{P}\int_{-1}^{1} Y_1'(\xi)\coth \lambda(x - \xi)\ d\xi = -\frac{2\lambda t}{\beta\pi} \mathcal{P}\int_{-1}^{1} \xi \coth \lambda(x - \xi) d\xi$$

and, after an integration by parts,

$$\bar{u}_1 = -\frac{2t}{\beta\pi}\mathcal{P}\int_{-1}^{1} \log_e \left|\sinh \lambda(x - \xi)\right| d\xi +$$

$$\frac{2t}{\beta\pi}\Big[\log_e \left|\sinh \lambda(x - 1)\right| + \log_e \left|\sinh \lambda(x + 1)\right|\Big]$$

Writing the hyperbolic sine in terms of exponentials, making a change of variable, and performing the elementary integrations gives

$$\bar{u}_1 = -\frac{t}{\beta\lambda\pi}\mathcal{P}\int_{a}^{b} \frac{\log_e |w|}{w - 1} dw + \frac{2t}{\beta\pi}\Big[\log_e \left|\sinh \lambda(x - 1)\right| +$$

$$\log_e \left|\sinh \lambda(x + 1)\right| - 2\lambda x + 2 \log_e 2\Big]$$

where

$$w = 1 - e^{-2\lambda(x-\xi)}$$

$$a = 1 - e^{-2\lambda(x+1)}$$

$$b = 1 - e^{-2\lambda(x-1)}$$

The Spence integral is defined by

$$Rl(x) = \int_1^x \frac{\log_e |x'|}{x' - 1} dx'$$

Thus with the use of this integral and equation (25) the tunnel-wall correction to the velocity is

$$\Delta u_1 = \frac{t}{\beta\lambda\pi}\left[Rl(a) - Rl(b)\right] + \frac{2t}{\beta\pi}\left[\log_e \left|\sinh \lambda(x - 1)\right| + \log_e \left|\sinh \lambda(x + 1)\right| - 2\lambda x + 2 \log_e 2 - 2 - x \log_e \left|\frac{1 - x}{1 + x}\right|\right] \tag{26}$$

The Spence integral is tabulated for positive values of x in reference 11. The limit b is negative for all values of x for $\lambda \geqq 0$. However it is easily shown that the value of the Spence integral for negative values of the argument is given by the relation

$$Rl(-x) = \frac{1}{2} Rl(x^2) - Rl(x) + \frac{3}{2} Rl(0)$$

The first-order solution for the tunnel-wall correction in series form is found from equation (26) as

$$\Delta u_1 = -\frac{2t}{\beta\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^{2n}\lambda^{2n}B_n}{(2n)!} \int_{-1}^{1} \xi(x - \xi)^{2n-1} d\xi$$

$$= \frac{t}{\beta\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^{2n}\lambda^{2n}B_n}{(2n)!n(2n + 1)} \left[(x - 1)^{2n}(x + 2n) - (x + 1)^{2n}(x - 2n)\right] \tag{27}$$

Similar series expressions for $\overline{\phi}_1$, ϕ_1, and their derivatives are easily found. Then, with the use of these first-order solutions, the functions $\overline{F}(\xi)$ and $F(\xi)$ may be evaluated. The integral involving the function $\overline{f}(\xi)$ is very small and may be neglected. The tunnel-wall correction to the second order is then found from equation (24).

RESULTS AND DISCUSSION

The first-order constriction correction to the x-component of velocity at the surface of a symmetric parabolic-arc airfoil in a wind tunnel is shown in figures 2 and 3 for several ratios of tunnel height to chord. It appears, at least for the parabolic-arc airfoil, that the error made in neglecting the variation of interference-velocity correction along the chord is small provided the chord is much less than the tunnel height and the Mach number is small. For these cases, the constant term of equation (23) gives a good value for the correction. However, these are just the cases for which the correction is so small that it is often neglected. For cases where the correction is significant (large Mach numbers and/or small ratios of tunnel height to chord), the error made in neglecting the variation of velocity correction along the chord may be 10 to 15 percent. Comparison of the correction (fig. 2) given by equation (26) with that given by equation (23) indicates good convergence of the series solution. The proper value of the correction is zero at a stagnation point. The fact that the correction does not approach zero at the stagnation points gives another example of the inability of this approximate theory to represent adequately the flow in these regions.

A series for the first-order correction to the velocity is given by equation (23); the first term gives the often-quoted result that the tunnel-wall correction varies as $1/\beta^3$, whereas succeeding terms contain other powers of β. The constriction correction to the second-order for the parabolic-arc airfoil at midchord is shown in figure 3. It is of interest to note that the correction, to the second order, at the midchord varies with Mach number approximately as $1/\beta^3$, the second-order correction being about 20 percent higher for $t = 0.1$ and about 40 percent higher for $t = 0.2$. It therefore appears that the second-order correction should be considered for test conditions where the first-order correction is significant.

The constriction corrections commonly employed to correct tunnel force or pressure data are applied to the stream velocity and hence to the dynamic pressure, Mach number, and so forth since the correction has a constant value over the chord. On the other hand, the corrections developed herein vary over the chord of the airfoil and consequently

they must be employed locally to correct the surface velocity or pressures. These second-order constriction corrections are more accurate than those normally employed but their calculation and use entails more labor.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., February 2, 1951

REFERENCES

1. Glauert, H.: Wind Tunnel Interference on Wings, Bodies and Airscrews. R. & M. No. 1566, British A.R.C., 1933.

2. Allen, H. Julian, and Vincenti, Walter G.: Wall Interference in a Two-Dimensional-Flow Wind Tunnel, with Consideration of the Effect of Compressibility. NACA Rep. 782, 1944.

3. Lamla, Ernst: On the Symmetrical Potential Flow of Compressible Fluid past a Circular Cylinder in the Tunnel in the Subcritical Zone. NACA TM 1018, 1942.

4. Emmons, Howard W.: Flow of a Compressible Fluid past a Symmetrical Airfoil in a Wind Tunnel and in Free Air. NACA TN 1746, 1948.

5. Epstein, H. T., and Albers, L. U.: The Effects of Compressibility on the Two-Dimensional Subsonic Wind-Tunnel Constriction Correction. Jour. Aero. Sci., vol. 15, no. 3, March 1948, pp. 144-150.

6. Harder, Keith C., and Klunker, E. B.: On a Source-Sink Method for the Solution of the Prandtl-Busemann Iteration Equations in Two-Dimensional Compressible Flow. NACA TN 2253, 1950.

7. Kaplan, Carl: The Flow of a Compressible Fluid past a Curved Surface. NACA Rep. 768, 1943.

8. Kaplan, Carl: On the Particular Integrals of the Prandtl-Busemann Iteration Equations for the Flow of a Compressible Fluid. NACA TN 2159, 1950.

9. Whittaker, E. T., and Watson, G. N.: A Course of Modern Analysis. American ed., The Macmillan Co., 1948, p. 136.

10. Dwight, Herbert Bristol: Tables of Integrals and Other Mathematical Data. Rev. ed., The Macmillan Co., 1947, p. 146.

11. Powell, E. O.: An Integral Related to the Radiation Integrals. Phil. Mag., ser. 7, vol. 34, no. 236, Sept. 1943, pp. 600-607.

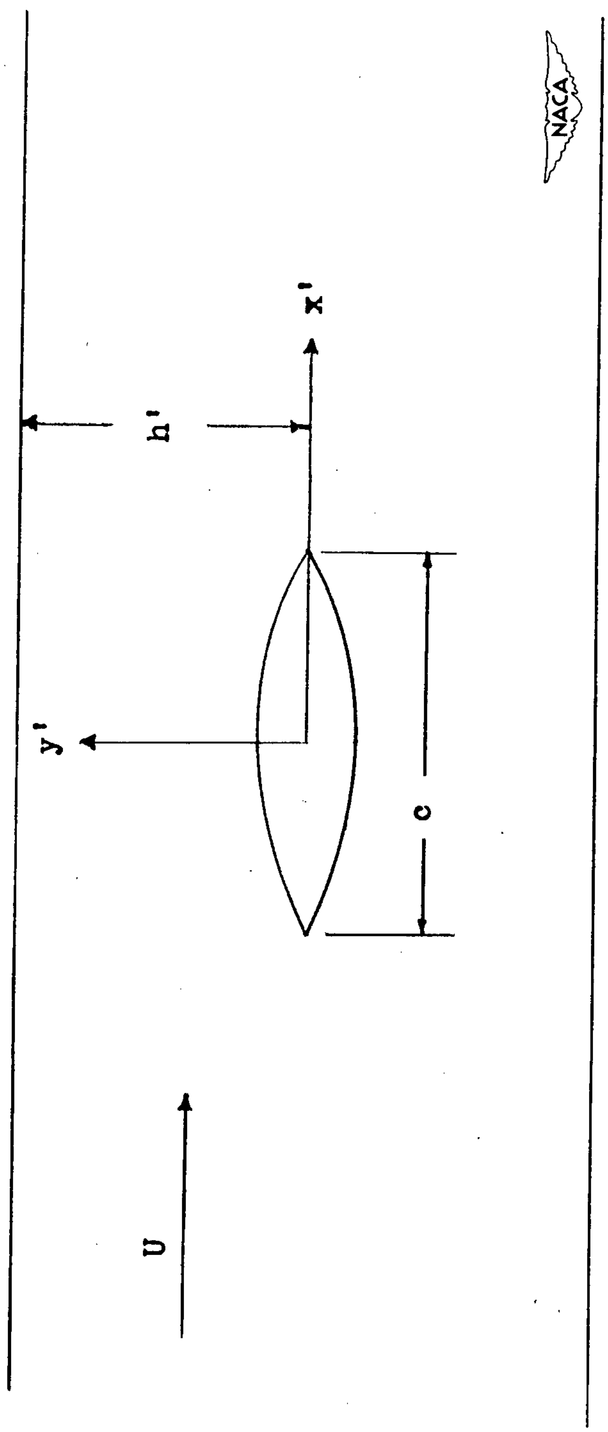
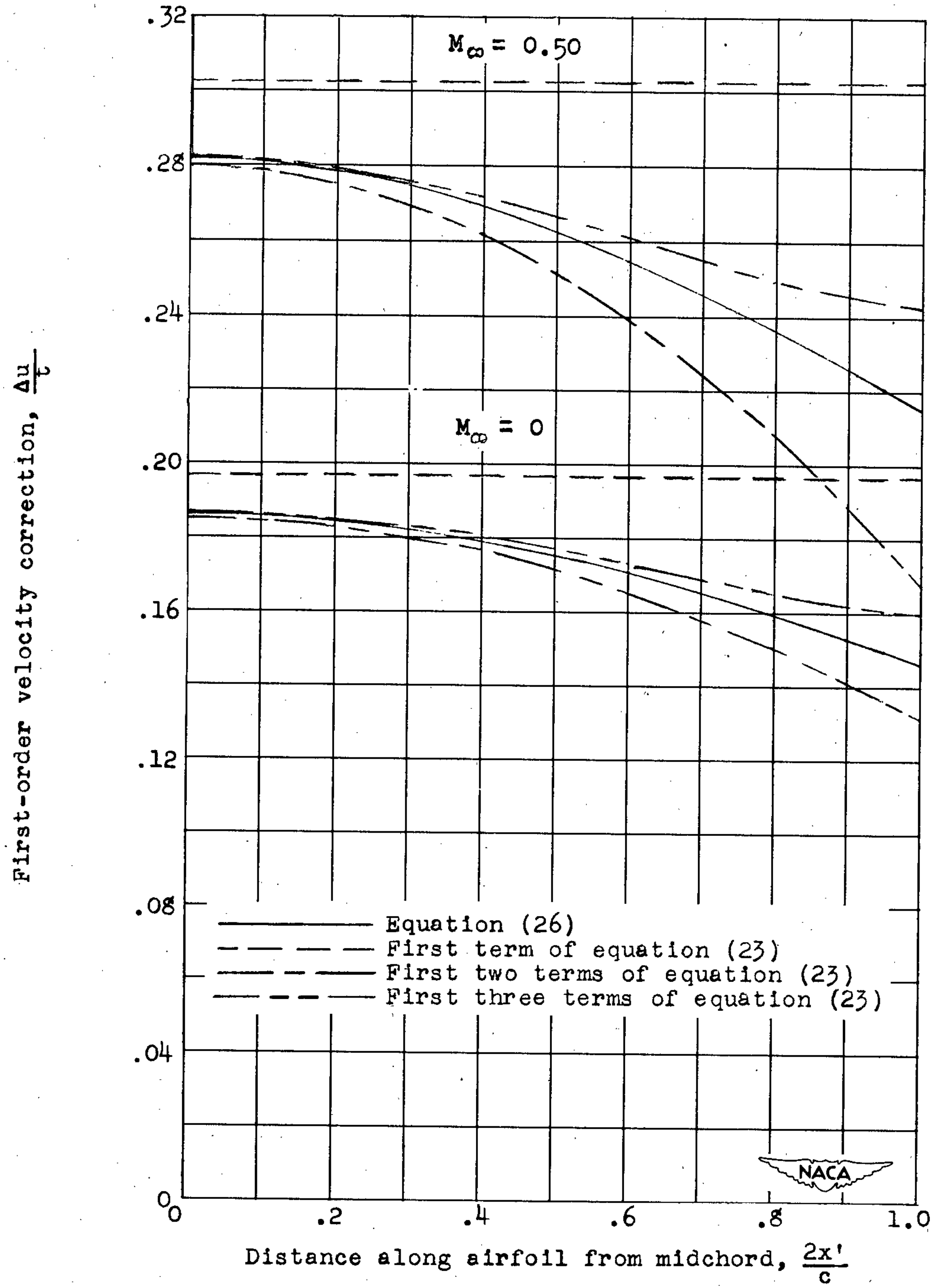
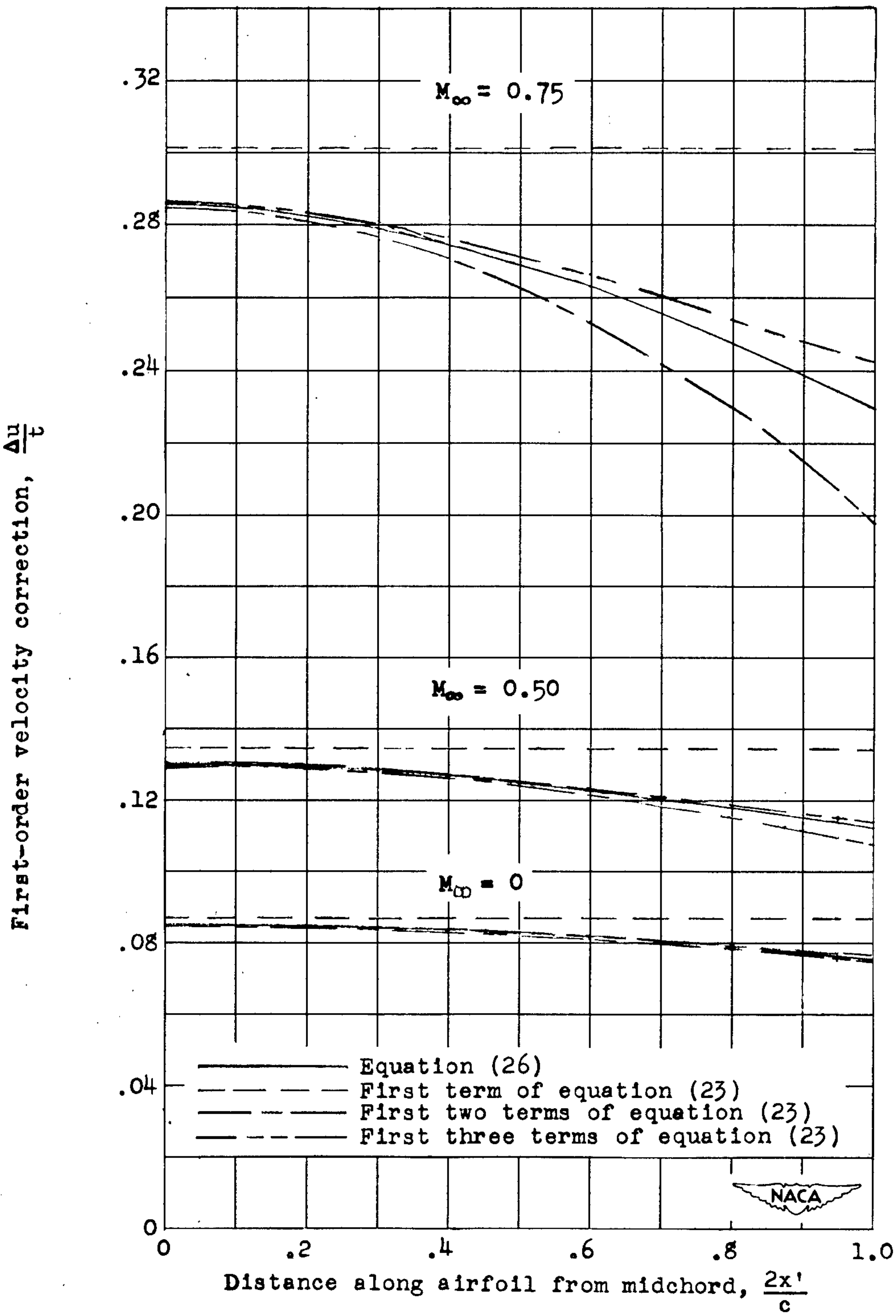


Figure 1.- Body in a two-dimensional channel.



(a) $\frac{2h'}{c} = \frac{4}{3}$.

Figure 2.- Comparison of closed-form solution (equation (26)) with series solution (equation (23)) for parabolic-arc airfoil.



(b) $\frac{2h'}{c} = 2.$

Figure 2.- Concluded.

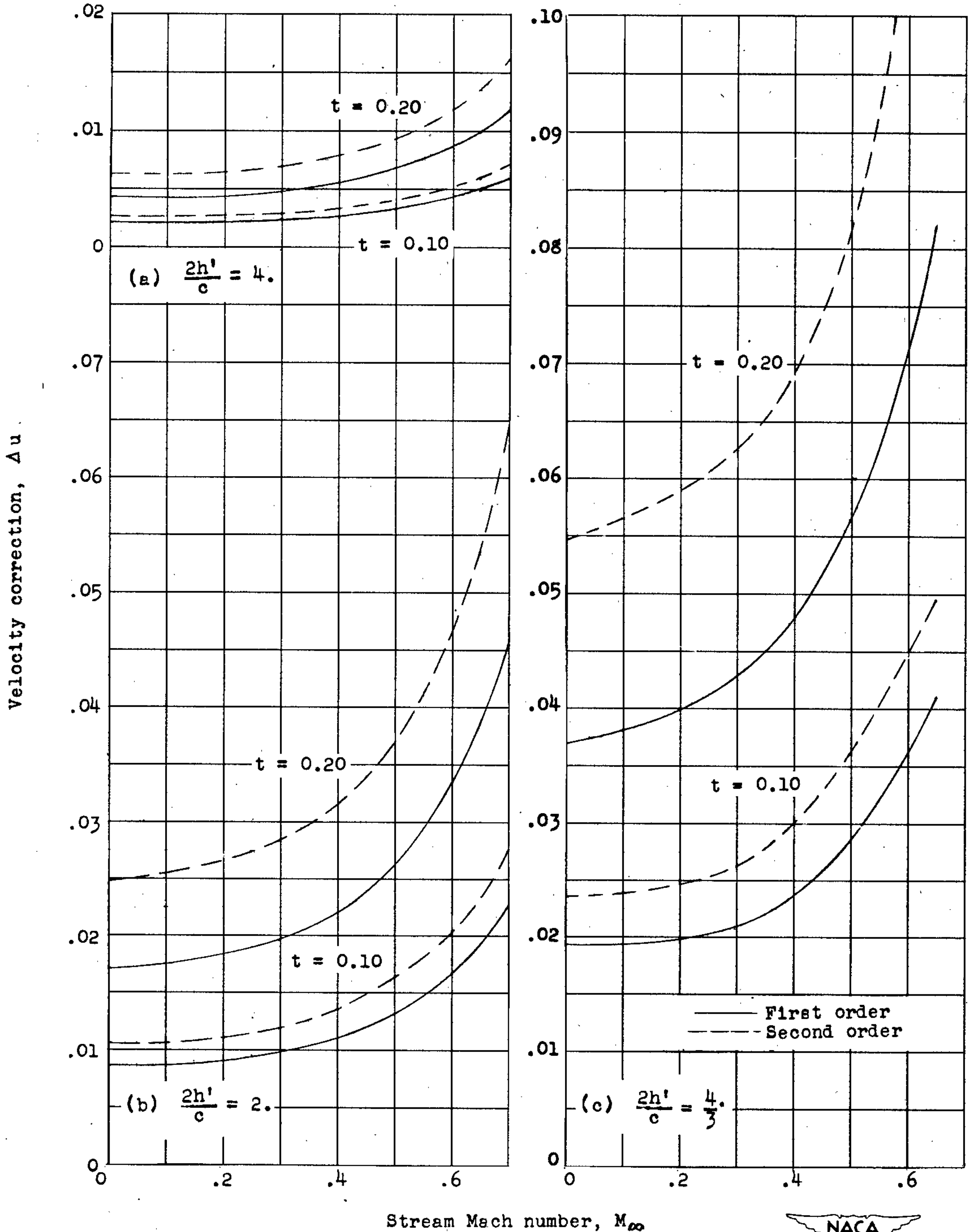


Figure 3.- Variation of first- and second-order velocity correction at the midchord with Mach number.

Flow, Subsonic 1.1.2.1

On the Second-Order Tunnel-Wall-Constriction Corrections in Two-Dimensional Compressible Flow.

By E. B. Klunker and Keith C. Harder

NACA TN 2350
April 1951

(Abstract on Reverse Side)

Wing-Section Theory 1.2.1.1

On the Second-Order Tunnel-Wall-Constriction Corrections in Two-Dimensional Compressible Flow.

By E. B. Klunker and Keith C. Harder

NACA TN 2350
April 1951

(Abstract on Reverse Side)

Research Technique, Corrections 9.2.1

On the Second-Order Tunnel-Wall-Constriction Corrections in Two-Dimensional Compressible Flow.

By E. B. Klunker and Keith C. Harder

NACA TN 2350
April 1951

(Abstract on Reverse Side)

Abstract

Solutions of the first- and second-order Prandtl-Busemann iteration equations are obtained for the flow past thin, sharp-nose, symmetric, two-dimensional bodies in closed channels. With the use of these solutions an expression is derived for the tunnel-wall interference. The tunnel-wall correction for a parabolic-arc airfoil is calculated to indicate the effects of compressibility, ratio of the tunnel height to the airfoil chord, and airfoil thickness coefficient. It appears that, for cases where the tunnel-wall corrections are significant, both the second-order effects and the variation of the correction along the chord should be considered.

Abstract

Solutions of the first- and second-order Prandtl-Busemann iteration equations are obtained for the flow past thin, sharp-nose, symmetric, two-dimensional bodies in closed channels. With the use of these solutions an expression is derived for the tunnel-wall interference. The tunnel-wall correction for a parabolic-arc airfoil is calculated to indicate the effects of compressibility, ratio of the tunnel height to the airfoil chord, and airfoil thickness coefficient. It appears that, for cases where the tunnel-wall corrections are significant, both the second-order effects and the variation of the correction along the chord should be considered.

Abstract

Solutions of the first- and second-order Prandtl-Busemann iteration equations are obtained for the flow past thin, sharp-nose, symmetric, two-dimensional bodies in closed channels. With the use of these solutions an expression is derived for the tunnel-wall interference. The tunnel-wall correction for a parabolic-arc airfoil is calculated to indicate the effects of compressibility, ratio of the tunnel height to the airfoil chord, and airfoil thickness coefficient. It appears that, for cases where the tunnel-wall corrections are significant, both the second-order effects and the variation of the correction along the chord should be considered.

NACA TN 2350